1 Math

$\dot{y} + f(x)y = g(x) \rightarrow$	$\frac{d}{dx}\left(ye^{\int f(x)\mathrm{d}x}\right) = g(x)e^{\int f(x)\mathrm{d}x}$
$\sum_{n=0}^{N} x^n \rightarrow$	$\frac{1-x^{N+1}}{1-x}$
$\sum_{n=-j}^{j} e^{n\eta} \rightarrow$	$\frac{\sinh((j+1/2)\eta)}{\sinh(\eta/2)}$
Gaussian Integral with cosine	$\int_{-\infty}^{\infty} \exp(-\alpha^2 x^2)\cos(\beta x)\mathrm{d}x = \frac{\sqrt{\pi}}{\alpha}\exp(-\frac{\beta^2}{4\alpha^2})$
Ellipses	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad r = \frac{p}{1+e\cos(\theta)} \quad p = (1-e^2)a$ $e = \sqrt{1-\frac{b^2}{a^2}} \quad F = ea \quad \text{Area} = \pi ab$
Cylindrical Laplacian	$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$
Spherical Laplacian	$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$

2 Classical Mechanics

Lagrange's Equations	$L = T - U$ $\frac{d}{dt}(\partial_{\dot{x}_i}L) - \partial_{x_i}L - \sum_k \lambda_k \partial_{x_i} f_k = 0$
Lagrange Multipliers	$f_k = 0 = f_k(x_i, x_j, \dot{x}_i, \ldots)$
Kepler's Laws	$T^2/R^3 = Constant$
Linear form of coupled ODE's	$A\mathbf{x} = \lambda\mathbf{x} = \ddot{\mathbf{x}} \quad \Rightarrow \quad \sqrt{\lambda} = \omega$
String Wave Equation	$\tau\partial_{xx}\psi - \mu\partial_{tt}\psi = 0$
Effective Spring Constant	$K = \frac{kL}{dx} \quad F = Kds$
Center of Mass	$\mathbf{R}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$
Moment of Inertia	$\mathsf{I}_{ij} = \int \rho(\delta_{i,j}x_k x_k - x_i x_j)\mathrm{d}V$
Generalized Parallel Axis Thm	$\mathsf{I}_{ij} = \mathsf{J}_{ij} - M(\delta_{i,j}a_k a_k - a_i a_j)$
Rotational Kinetic Energy	$T_{rot} = \frac{1}{2}\mathsf{I}_{i,j}\omega_i\omega_j$
Torque Equation of Motion	$N_1 = \mathsf{I}_1\frac{\partial}{\partial t}\omega_1 - (\mathsf{I}_3 - \mathsf{I}_2)\omega_3\omega_2$

3 Electricity and Magnetism

EM Boundary Conditions	$\Delta D_\perp = \sigma_f \quad \Delta E_\parallel = 0$ $\Delta B_\perp = 0 \quad \Delta H_\parallel = K$
Cylindrical Wave Equation Solution (no z)	$\Phi(\rho,\theta) = (a_0 + b_0\ln(\rho))(c_0 + d_0\phi)$ $+\sum_{n=1}^{\infty}(A_n\rho^n + B_n\rho^{-n})\sin(n\phi)$ $+\sum_{n=1}^{\infty}(C_n\rho^n + D_n\rho^{-n})\cos(n\phi)$
Spherical Wave Equation Solution	$\Phi(r,\theta) = \sum_{l=0}^{\infty}\left[A_l r^l + B_l r^{-(l+1)}\right]P_l(\cos(\theta))$ $\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[A_{lm}r^l + B_{lm}r^{-(l+1)}\right]Y_{lm}(\theta,\phi)$
Waveguide Wave Equations(TM or TE)	$\nabla_\perp^2 E_z = (\omega^2/c^2 - k^2)E_z$ $\nabla_\perp^2 B_z = (\omega^2/c^2 - k^2)B_z$
Electric Dipole Moment	$\mathbf{p} = \int \mathbf{x}\rho(\mathbf{x})\mathrm{d}V$
Electric Field from **p**	$\mathbf{E}(\mathbf{x}) = \frac{3\hat{\boldsymbol{n}}(\hat{\boldsymbol{n}}\cdot\mathbf{p}) - \mathbf{p}}{4\pi\epsilon_0\lvert\mathbf{x}\rvert^3}$
Magnetic Dipole Moment	$\mathbf{m} = \frac{1}{2}\int \mathbf{x}\times\mathbf{j}(\mathbf{x})\mathrm{d}V$
B Field from **m**	$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{3\hat{\boldsymbol{n}}(\hat{\boldsymbol{n}}\cdot\mathbf{m}) - \mathbf{m}}{\lvert\mathbf{x}\rvert^3}$
Electric Dipole Total Power Radiated	$P_{Edip} = \frac{2}{3}\frac{\lvert\ddot{p}\rvert^2}{4\pi\epsilon_0 c^3}$
Magnetic Dipole Total Power Radiated	$P_{Bdip} = \frac{2}{3}\frac{\lvert\ddot{m}\rvert^2}{4\pi\epsilon_0 c^5}$
curl(curl**E**)	$\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E}$
Poynting Vector	$\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$ $\mathbf{S}_{av} = \mathrm{Re}(\mathbf{E}\times\mathbf{B}^*/\mu_0)$
Poynting Vector for Single Radiating Charge	$\mathbf{S}_{av} = \frac{Q^2a^2\sin^2(\theta)}{16\pi^2\epsilon_0 c^3 r^2}\hat{\boldsymbol{r}}$
Vector Potential	$\mathbf{A}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{j}(\mathbf{x}, t - \lvert\mathbf{x}-\mathbf{x}'\rvert/c)}{\lvert\mathbf{x}-\mathbf{x}'\rvert}\mathrm{d}V'$
Circuit Voltage Equivalents	$V = \{Q/C,\ RdQ/dt,\ Ld^2Q/dt^2\}$
Transformer Voltage Equation Self (Mutual through 2 from 1) Inductance	$V_1 + L_1 dI_1/dt + M_{12}dI_2/dt = 0$ $L_{1(2)} = \frac{N_{1(,2)}\Phi_{1(,2)}}{I_1}$
Energy Density (and Total Energy)	$u = \{\epsilon_0 E^2/2,\ B^2/2\mu_0\}(E = \{LI^2/2,\ CV^2/2\})$

4 Quantum Mechanics

Schrodinger Equation	$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \hat{H}\psi(\mathbf{r},t) \quad \psi(\mathbf{r},t) = \exp(-\frac{i\hat{H}t}{\hbar})\psi(\mathbf{r},t=0)$
Heisenberg Picture	$A(t) = U^*(t)AU(t) \quad U(t) = e^{-i\hat{H}t/\hbar} \quad \frac{\partial \hat{A}}{\partial t} = \frac{i}{\hbar}[\hat{H},\hat{A}]$
Differential Cross Section	$\frac{d\sigma}{d\Omega} = \|f_k(\theta,\phi)\|^2$
Scattering Amplitude (Born Approximation)	$f_k(\theta,\phi) = -\frac{m}{2\pi\hbar}\int \mathrm{d}^3x' \exp(i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x})V(\mathbf{x}')$ $f_k(\theta,\phi) = -\frac{2m}{\hbar^2 K}\int_0^\infty \mathrm{d}r\, r\, V(r)\sin(Kr)$
Scattering Amplitude (Partial Wave)	$f_k(\theta) = \sum_l (2l+1)a_l(k)P_l(\cos(\theta)) \quad a_l(k) = \frac{1}{k}e^{i\delta_l(k)}\sin(\delta_l(k))$ $\sigma_T = \frac{4\pi}{k^2}\sum_l (2l+1)\|\sin(\delta_l(k))\|^2$
Optical Theorem	$\sigma_T = \frac{4\pi}{k}\mathrm{Im}(f(0))$
Fourier Transform of Yukawa Potential	$V(\mathbf{x}) = \frac{\exp(-\mu\|\mathbf{x}\|)}{\|\mathbf{x}\|} \quad \widetilde{V}(k) = \frac{4\pi}{\mu^2 + \|k\|^2}$
Hydrogen Wave Functions (1s, 2s, 2p)	$\psi_{100} = \frac{2}{a_0^{3/2}}\exp(-r/a_0)Y_{00} \quad a_0 = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2}$ $\psi_{200} = \frac{2}{(2a_0)^{3/2}}\left(1 - \frac{r}{2a_0}\right)\exp(-r/2a_0)Y_{00}$ $\psi_{21m} = \frac{1}{\sqrt{3}(2a_0)^{3/2}}\frac{r}{a_0}\exp(-r/2a_0)Y_{1m}$
$l = 0, 1$ Spherical Harmonics	$Y_{00} = 1/\sqrt{4\pi} \quad Y_{10} = \frac{1}{2}\left(\frac{3}{\pi}\right)^{1/2}\cos(\theta)$ $Y_{1,\pm 1} = \mp\frac{1}{2}\left(\frac{3}{2\pi}\right)^{1/2}\sin(\theta)\exp(\pm i\phi)$
Dipole Radiation Selection Rules	$\Delta l = \pm 1 \quad \Delta m = \pm 1, 0$
Time-Independent Nondegenerate Perturbation Theory	$E_n = E_n^{(0)} + H'_{nn} + \sum_{i\neq n}\frac{\|H'_{ni}\|^2}{E_n^{(0)} - E_i^{(0)}}$ $\varphi_n = \varphi_n^{(0)} + \sum_{i\neq n}\frac{H'_{in}}{E_n^{(0)} - E_i^{(0)}}\varphi_i^{(0)} \quad H'_{in} \equiv \langle i\|H'\|n\rangle$
Time Dependent Perturbation Theory Long Time Limit Rate	$P_{m\to n} = 1/\hbar^2\left\|\int_{t_o}^t \exp(i(E_n - E_m)t/\hbar)\langle n\|V(t)\|m\rangle \mathrm{d}t'\right\|^2$ $\Gamma_{m\to n} = 2\pi/\hbar^2\delta(E_n - E_m)\|\langle n\|V_o\|m\rangle\|^2$
Fermi Golden Rule (#2)	$\Gamma_{m\to n} = \frac{2\pi}{\hbar}\mathcal{D}(E_n)\|\langle n\|V_o\|m\rangle\|^2$
Variational Principle	$E_o \leq \langle\psi\|H\|\psi\rangle,\ \langle\psi\|\psi\rangle = 1$
Lowering Operator $(\hat{x},\hat{p})$	$\hat{a} = \hat{x}\sqrt{m\omega/2\hbar} + \frac{i\hat{p}}{\sqrt{2m\hbar\omega}} \quad \hat{a}\|n\rangle = n^{1/2}\|n-1\rangle$
Angular Momentum $\pm$ Operators	$\hat{L}_\pm\|l,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\ \|l,m\pm 1\rangle$
Radial Schrodinger Equation	$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \frac{\hbar^2 l(l+1)}{2mr^2} + V(r)\right]U(r) = E\,U(r) \quad U(0) = 0$

5 Statistical Mechanics

Quantum Length and Concentration	$\lambda_Q = \frac{h}{\sqrt{2\pi MT}} \quad n_Q = \left(\frac{MT}{2\pi\hbar^2}\right)^{3/2}$
Thermodynamic Identity	$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N$
Free Energy	$F = U - TS = -T\ln(Z)$
Gibbs Free Energy	$G = \mu N = U + PV - ST$
Enthalpy	$H = U + PV$
Thermo Function (Ω)	$\Omega = -PV = -T\sum_{ASN}\ln(1+\exp(\frac{N\mu-\varepsilon}{T}))$
Heat Capacities	$C_V = \left.\frac{dU}{dT}\right\vert_V \quad C_P = \left.\frac{dU}{dT}\right\vert_P$
Partition Function	$Z = \sum\exp(-E/T)$
Gibbs' Partition Function	$Z = \sum\exp((\mathbf{J}\cdot\mathbf{x} - E)/T)$
Grand Cononical Sum Sum for bosons	$\mathcal{Z} = \sum_{ASN}\exp[(\mu - \varepsilon_{s(N)})/T]$ $\mathcal{Z} = \frac{1}{1-\exp[(\mu-\varepsilon)/T]}$
Absolute Activity	$\lambda = \exp(\mu/T)$
Occupation factor	$f(\varepsilon) = \langle N(\varepsilon)\rangle = \lambda\frac{d}{d\lambda}\log(\mathcal{Z})$
Occupation of Fermions $f(\varepsilon)$	$f(\varepsilon) = \frac{1}{\exp(\varepsilon-\mu/T)+1}$
Occupation of Bosons $f(\varepsilon)$	$f(\varepsilon) = \frac{1}{\exp(\varepsilon-\mu/T)-1}$
Non-relativistic Density of States	$\mathcal{D}(\varepsilon) = \frac{\mathrm{d}^D x\ \mathrm{d}^D p}{(2\pi\hbar)^D}$
Expectation value using $f(\varepsilon)$and$\mathcal{D}(\varepsilon)$	$\langle X\rangle = \int\mathcal{D}(\varepsilon)f(\varepsilon)X(\varepsilon)\mathrm{d}\varepsilon$
Ideal Gas Relations	$U = \frac{3}{2}PV \quad \mu = T\ln(\frac{n}{n_Q}) \quad Z_1 = \frac{n_Q}{n}$ $S = N\left[\ln(\frac{n_Q}{n} + \frac{5}{2})\right]$
Clausius-Clapeyron Relation	$\frac{dP}{dT} = \frac{s_g - s_l}{v_g - v_l} \quad s_i = S_i/N_i,\ v_i = V_i/N_i$