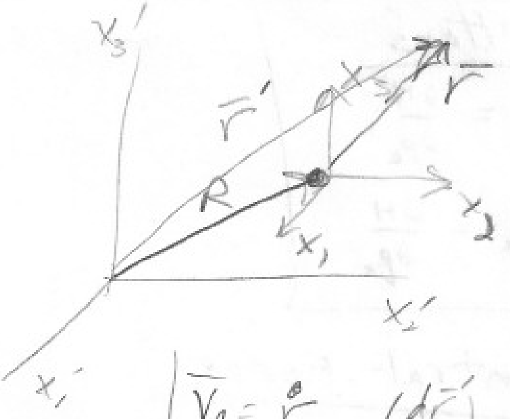
CM

$$\left(\frac{d\bar{Q}}{dt}\right)_{fixed} = \left(\frac{d\bar{Q}}{dt}\right)_{rotating} + \bar{\omega}\times\bar{Q}$$



$$\bar{V}_f = \bar{V} + \bar{v}_r + \bar{\omega}\times\bar{r}$$

ie: $$\left(\frac{d\bar{r}'}{dt}\right)_{fixed} = \left(\frac{d\bar{R}}{dt}\right)_{fixed} + \left(\frac{d\bar{r}}{dt}\right)_{rot} + \bar{\omega}\times\bar{r}$$

$$\bar{V}_f = \dot{r}'_f = \left(\frac{d\bar{r}'}{dt}\right)_{fixed}$$

$$\bar{V} = \dot{R}_f = \left(\frac{dR}{dt}\right)_{fixed}$$

$$\bar{V}_r = \dot{r}_r = \left(\frac{d\bar{r}}{dt}\right)_{rot}$$

$$F = m\left(\frac{d\bar{v}_f}{dt}\right)_{fixed}$$

$$F_{eff} = \bar{F}_{fix} - m\ddot{\bar{R}}_f - m\dot{\bar{\omega}}\times\bar{r} - m\bar{\omega}\times(\bar{\omega}\times\bar{r}) - 2m\bar{\omega}\times\bar{v}_r$$

translational acceleration of frame; angular acceleration of frame; centrifugal; coriolis

Earth: $$F_{eff} = \bar{S} + \underbrace{m g_0 - m\bar{\omega}\times(\bar{\omega}\times(\bar{r}+\bar{R}))}_{m\bar{g}} - 2m\bar{\omega}\times\bar{v}_r$$

pushes out; coriolis

$$T_{rot} = \frac{1}{2}\sum_\alpha m_\alpha(\bar{\omega}\times\bar{r}_\alpha)^2 = \frac{1}{2}\sum_{ij} I_{ij}\omega_i\omega_j = \frac{1}{2}\bar{\omega}\cdot\bar{L} = \frac{1}{2}\bar{\omega}\cdot\bar{\bar{I}}\cdot\bar{\omega}$$

$$I_{ij} = \int_V \rho(\bar{r})\left(\delta_{ij}\sum_k x_k^2 - x_i x_j\right) d^3x \; : \quad I = \int\rho(r)\, x_\perp^2\, d^3x, \quad I = \int r^2\, dm$$

note: $I_{ij} = I_{ji}$

or $I_{ij} = \sum_\alpha m_\alpha\left(\delta_{ij}\sum_k - x_{\alpha,i}\,x_{\alpha,j}\right)$ for point masses

Parallel Axis: $I_{ij} = J_{ij} - M(a^2\delta_{ij} - a_i a_j)$

COM; shifted; $\sum_k a_k$

$$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = N_1 \quad \text{and cyclic}$$

$$\bar{L} = \bar{r}\times\bar{p} = I\omega$$

$$\bar{N} = \bar{r}\times\bar{F} = \bar{r}\times\dot{\bar{p}} = \dot{\bar{L}}$$

$$\dot{p}_j = \frac{\partial L}{\partial q_j}, \quad p_j = \frac{\partial L}{\partial \dot{q}_j}$$

Hamiltons

$$\dot{q}_k = \frac{\partial H}{\partial p_k}$$

$$-\dot{p}_k = \frac{\partial H}{\partial q_k}$$

Central Force

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$p_\theta = l = \mu r^2 \dot{\theta} = \text{constant} = \frac{\partial L}{\partial \dot{\theta}}$$

$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$$

$$\rightarrow \dot{r} = \pm\sqrt{\frac{2}{\mu}(E-U) - \frac{l^2}{\mu^2 r^2}}$$

$$d\theta = \frac{d\theta}{dt}\frac{dt}{dr} = \frac{\dot{\theta}}{\dot{r}}dr$$

$$\theta(r) = \int \frac{\pm\left(\frac{l}{r^2}\right)dr}{\sqrt{2\mu\left(E - U - \frac{l^2}{2\mu r^2}\right)}}$$

let $u = \frac{1}{r}$

$$\frac{d^2 u}{d\theta^2} + u = -\frac{\mu}{l^2 u^2} F(1/u)$$

$$\rightarrow \boxed{\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{l^2} F(r)}$$

$$\frac{l^2}{2\mu r^2} = \frac{1}{2}\mu r^2 \dot{\theta}^2 = U_c$$

$$F_c = -\frac{\partial U_c}{\partial r} = \frac{l^2}{\mu r^3} = \mu r \dot{\theta}^2$$

$$V(r) = U(r) + \frac{l^2}{2\mu r^2}$$

if $\alpha = \frac{l^2}{\mu k}$

$$\varepsilon = \sqrt{1 + \frac{2El^2}{\mu k^2}}$$

$$\frac{\alpha}{r} = 1 + \varepsilon\cos\theta$$

$\varepsilon > 1$ $E > 0$ Hyperbola

$\varepsilon = 1$ $E = 0$ Parabola

$0 < \varepsilon < 1$ $V_{min} < E < 0$ ellipse

$\varepsilon = 0$ $E = V_{min}$ circle

$\varepsilon < 0$ $E < V_{min}$ not allowed

$$a = \frac{\alpha}{1-\varepsilon^2} = \frac{k}{2|E|} \quad \text{semi-major}$$

$$b = \frac{\alpha}{\sqrt{1-\varepsilon^2}} = \frac{l}{\sqrt{2\mu|E|}} = \sqrt{\alpha a}$$

$$r_{min} = \frac{\alpha}{1+\varepsilon}$$

$$r_{max} = \frac{\alpha}{1-\varepsilon}$$

$$\tau^2 = \frac{4\pi^2\mu}{k}a^3$$

$k = Gm_1 m_2$ for gravity

Rigid Bodies

$$\bar{R} = \frac{1}{M}\sum_\alpha m_\alpha \bar{r}_\alpha = \frac{1}{M}\int \bar{r}\,dm$$

$$\bar{L} = \bar{R}\times\bar{P} + \sum_\alpha \bar{r}_\alpha \times \bar{p}_\alpha$$

$$T_{ROT} = \frac{1}{2}\sum_\alpha m_\alpha (\bar{\omega}\times\bar{r}_\alpha)^2 = \frac{1}{2}\sum_{i,j} I_{ij}\,\omega_i\omega_j\,,\quad I_{ij} = \sum_\alpha m_\alpha\left(\delta_{ij}\sum_k x_{\alpha,k}^2 - x_{\alpha,i}x_{\alpha,j}\right)$$

$$I_{ij} = \int \rho(\bar{r})\left(\delta_{ij}\sum x_k^2 - x_i x_j\right)dVol$$

$$L_i = \sum_j I_{ij}\,\omega_j = \sum_\alpha m_\alpha\left[r_\alpha^2\bar{\omega} - \bar{r}_\alpha(\bar{r}_\alpha\cdot\bar{\omega})\right] = \bar{\bar{I}}\cdot\bar{\omega}$$

so $T_{ROT} = \frac{1}{2}\bar{\omega}\cdot\bar{L} = \frac{1}{2}\bar{\omega}\cdot\bar{\bar{I}}\cdot\bar{\omega}$

$$I_{ij} = J_{ij} - M(a^2\delta_{ij} - a_i a_j)$$

Coupled oscillations

$$T = \frac{1}{2}\sum_{j,k} m_{jk}\,\dot{q}_j\dot{q}_k\,,\quad U = \frac{1}{2}\sum_{j,k} A_{jk}\,q_j q_k \qquad \text{with } A_{jk} = \left.\frac{\partial^2 U}{\partial q_j \partial q_k}\right|_{q_0}$$

(at equilibria)

$$\sum_j \left(A_{jk}q_j + m_{jk}\ddot{q}_j\right) = 0$$

$$\sum_j \left(A_{jk} - \omega_r^2 m_{jk}\right)a_{jr} = 0$$

$$\begin{vmatrix} A_{11} - \omega^2 m_{11} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & A_{nn}\,m_{nn} \end{vmatrix} = 0$$

Find eigenvectors with

ie: $(A_{11} - \omega_1^2 m_{11})a_{11} + (A_{21} - \omega_1^2 m_{21})a_{21} = 0$

SR

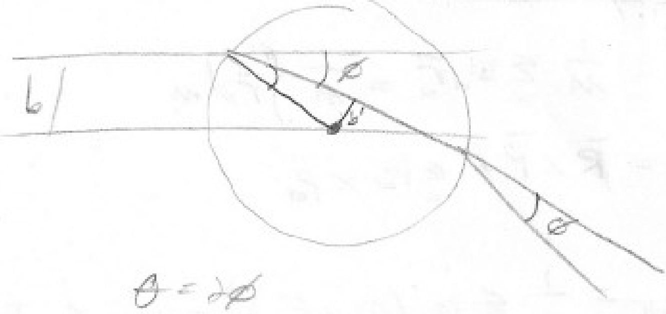
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}\qquad \beta = \frac{v}{c}\qquad \bar{p} = \gamma m\bar{u}\,,\quad m = \gamma m_0$$

$$T = \gamma mc^2 - mc^2 = (\gamma-1)mc^2$$

$$E = \gamma mc^2 = T + E_0$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix},\quad \begin{pmatrix} E' \\ p_x'c \\ p_y'c \\ p_z'c \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E \\ p_xc \\ p_yc \\ p_zc \end{pmatrix}$$

Scattering



$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\frac{db}{d\theta}$$

use conservation of E, L

$\theta = 2\phi$

$L = mbv_i$

$E + V_0 = E' + V_0'$